14th Joint European Thermodynamics Conference
Budapest, May 21–25, 2017

ON A NEW INTERPRETATION OF THE ENDOREVERSIBILITY HYPOTHESIS IN CURZON-AHLBORN-LIKE HEAT ENGINES.

Julian Gonzalez-Ayala[1,*], L. A. Arias Hernández[2] and F. Angulo-Brown [2]

[1]Departamento de Física Aplicada, Universidad de Salamanca, 37008 Salamanca, Spain
[2]Departamento de Física, Escuela Superior de Física y Matemáticas del Instituto Politécnico Nacional, 07738 Ciudad de México, Mexico
*jgonzalezayala@usal.es

ABSTRACT

The appearing of finite time thermodynamics (FTT) was a necessary step toward the analysis of more realistic heat engine (HE) models. As is well-known, the upper bound to the efficiency for any HE operating between two heat reservoirs is the known Carnot efficiency $\eta_C = 1-\tau$, being $\tau = T_c/T_h$ the ratio between the temperatures of the cold and hot reservoirs. Nevertheless, this efficiency is only achievable in a quasi-static process and with infinite operation time. Therefore, away from the performance of real devices.

One of the attempts to obtain more realistic efficiencies was the proposal of the endoreversible HE [1]. The endoreversible mode consists of an internal Carnot cycle operating between the temperatures T_{cw} and $T_{hw} \geq T_{cw}$, these thermal reservoirs are irreversibly coupled with two external reservoirs with temperatures T_c and $T_h \geq T_c$ (see Fig. 1). The principal result stemming from the seminal work by Curzon and Ahlborn in 1975 [1] about an endoreversible HE operating at maximum power (MP) output was the so called Curzon-Ahlborn (CA) efficiency $\eta_{CA} = 1-\sqrt{\tau}$. It was shown from experimental data that many energy plants operate with efficiencies noticeable closer to the η_{CA} value. Later on, additional elements have been incorporated into this model, such as a heat leak between the T_h and T_c heat reservoirs, the incorporation of irreversibilities in the internal Carnot cycle and the use of different heat transfer laws .

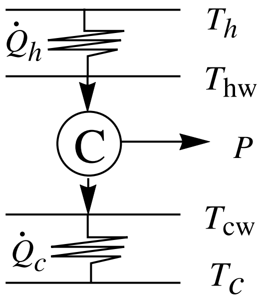

Figure 1: The scheme of an endoreversible HE of the Curzon-Ahlborn type.

Since the internal process is a Carnot cycle, then we have a relation between the entropy change in the hot and cold isothermal processes, $\Delta S_h = -\Delta S_c$ in order to have a zero entropy change for the internal cycle, this is the so-called endoreversible hypothesis. This equality allows us to give a relation between the thermal conductances or partial contact time, depending on the convention used in the model. Then, the optimization variables are the internal temperatures T_{cw} and T_{hw}. When there is no heat leak, the efficiency of the internal cycle is the efficiency of the engine. However, the Carnot core of the engine is always used. Here we discuss a new interpretation of the endoreversible hypothesis where the internal cycle is no longer a Carnot cycle.

In Ref. [2] we proposed a connection between the efficiency of HE's of the CA type operating at the MP regime and the efficiency of reversible cycles of the Otto and Joule-Brayton type (as the one depicted in Fig. 2) in the maximum work (MW) regime. This connection was made between a family of heat transfer laws and heat capacities depending on a power of the temperature. The heat transfer laws are as the following ones

$$\dot{Q}_h = \alpha\left(T_h^k - T_{hw}^k\right) \tag{1}$$

$$\dot{Q}_c = \beta\left(T_{cw}^k - T_c^k\right) \tag{2}$$

k is a real number and α and β are the heat conductances which for $k>0$ are positive constants and for $k<0$ are negative. On the other hand, the heat capacities are the following

$$C_{in} = aT^n \tag{3}$$

$$C_{out} = bT^n \tag{4}$$

where C_{in} and C_{out} are the heat capacities for the processes $1 \to 2$ and $3 \to 4$ (see Fig. 2), respectively, a and b are constant real numbers.

This connection allowed the obtention of analytical closed expressions close (and exact in some cases) to the FTT efficiencies at MP depending on the properties of the working substance and not only of the thermal gradient of the external heat reservoirs enclosing the HE, such as η_C and η_{CA}. Among the results stemming from this connection are upper and lower bounds for the endoreversible HE's efficiencies. Let us recall that in many

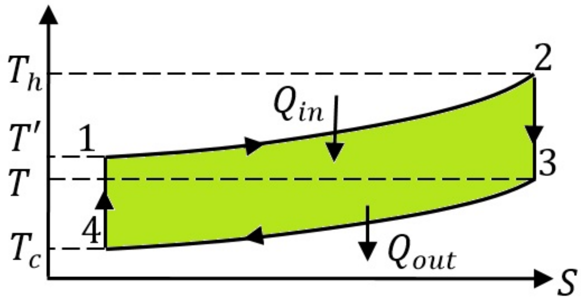

Figure 2: The scheme in the plane T–S of a HE formed with two adiabats and two processes with a positive heat capacity.

cases these calculations involve numerical methods and an analytical landmark depending on parameters of the working substance could result in an interesting path toward the design and modeling of these kind of engines.

With this in mind, we pursue the searching for analytical expressions for operation regimes involving entropy production such as the ecological function [3] or the Ω function [4], where the unavoidable losses due to the irreversible character of the HE's are considered. As it is well-known, reversible cycles have zero entropy change, then, the obtention of the reversible analog to the entropy production is not an obvious task.

Following the power-work connection we have obtained that the role of the irreversible dissipation in FTT endoreversible engines is played by the difference in areas between a reversible Otto-like HE and the Carnot cycle when both cycles operate between the same internal thermal reservoirs and with the same input heat. In this way, we have named this quantity a "geometrical dissipation" [5] since it is not a real dissipation and is not associated to an intrinsic property of a cycle, but it depends on a comparison between two cycles. In the maximization of power output and the maximum ecological function (ME) the resulting efficiencies depend on τ, the exponent k and the ratio of the conductances $\gamma \equiv \alpha/\beta$ and in the case of the reversible cycles they depend on τ, the exponent n and the ratio of the constants $\gamma' \equiv a/b$. We have taken the case $\gamma = \gamma'$ and for any given τ the efficiencies are bounded by the $\gamma = 0$ and $\gamma = \infty$ cases. The connection between both efficiencies is $k \to n+1$. In Fig. 3 we show the comparison between both efficiencies in the regime of $MP - MW$ and $ME - ME_r$, where ME_r is the maximum of the reversible ecological function. From this figure we can see that the behavior of both efficiencies (reversible and irreversible) are similar and in some cases the fitting is good. In Fig. 3 we can see also that in the range $k \in (-1, 3/4)$ the upper and lower branches of each χ–shaped curves are within the irreversible curves (dotted lines), and outside that range all the efficiencies are enclosed by the reversible curves (continuous lines) for both regimes.

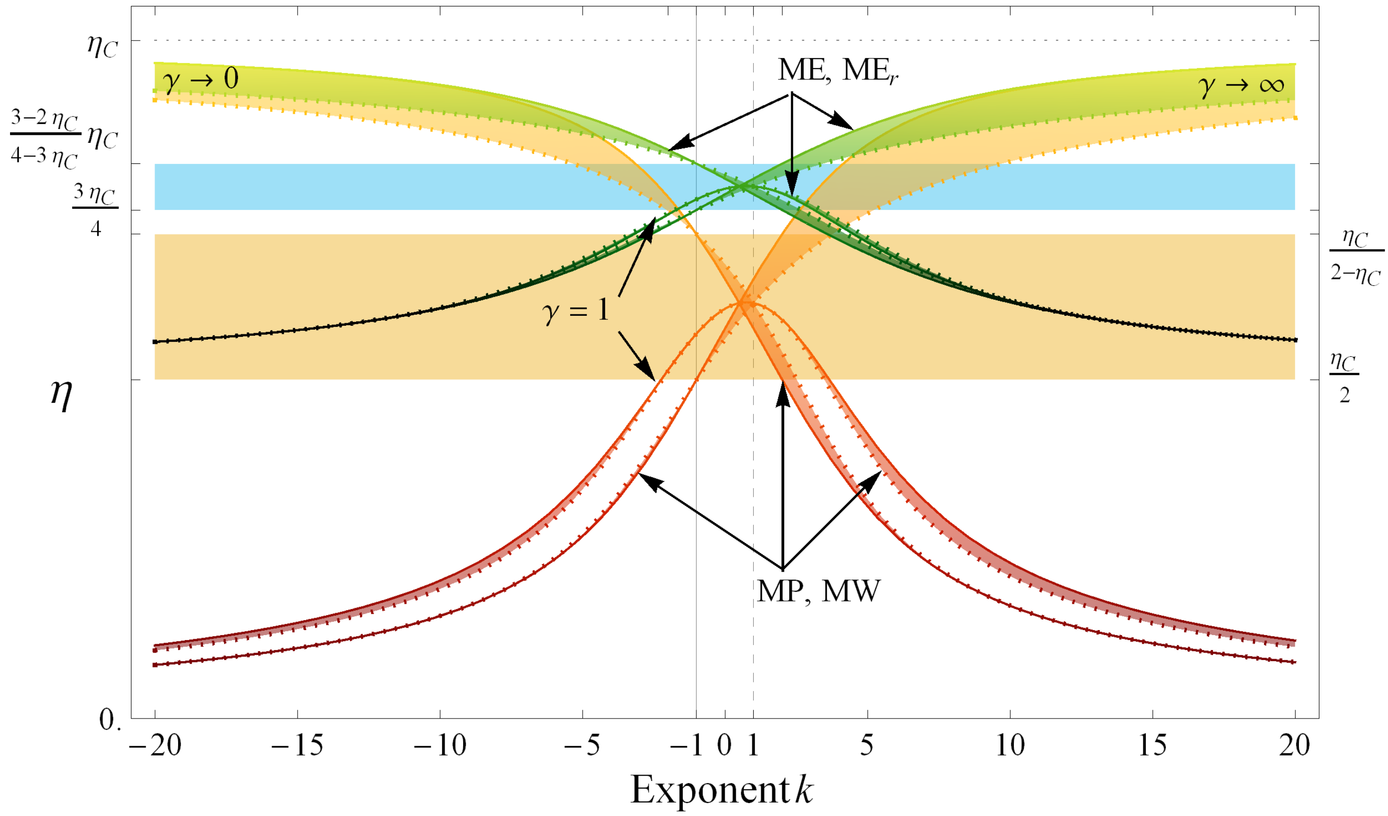

Figure 3: Efficiencies of the endoreversible HE model (dotted lines) for the MP and the ME regimes, and the reversible efficiencies (continuous lines) for the MW and ME_r regimes.

As can be seen in Fig. 3, there is a shaded area between the upper and lower bounds in the two regimes. For both regimes the lower branches present a better matching and as the efficiency get closer to η_C, the reversible cases have noticeable larger values, however, they never overcome the 10% error for the MP-MW case and the 6% error in the ME-ME_r case as can be seen in Fig. 4.

Our proposal to reinterpret the endoreversible hypothesis and explain the connection between CA-type HE's and Otto-like reversible cycles in the two regimes is that instead of considering a Carnot cycle, one can use any other reversible cycle (as will be shown later) or, in this case, an

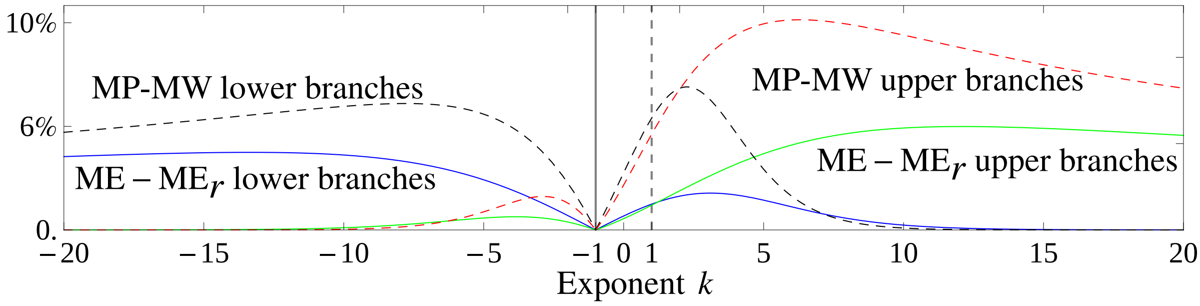

Figure 4: Percent error between the upper and lower branches of the *MP-MW* and *ME-ME$_r$* χ–shaped curves (see shaded areas between the pairs of reversible and endoreversible curves in Fig. 3).

irreversible Otto-type cycle, where the non-adiabatic processes are substituted by irreversible paths of minimum entropy generation. This procedure is as follows:

In Ref. [6] Anacleto and Ferreira showed that for a constant heat capacity working substance, the path of N auxiliary heat reservoirs under minimum entropy generation should follow the distribution given by $T_j = \sqrt{T_{j-1}T_{j+1}}$, this can be done for the heat capacities in Eqs. (3) and (4) and substitute the non-adiabatic paths in Fig. 2, then one obtains a minimum entropy generation approximation of the Otto-like cycle. It can be shown that with a few auxiliary reservoirs (about 32) one already have a good approximation to the reversible path. The relation between this cycle and the internal Carnot cycle is that if one substitute the non adiabatic paths for effective isotherm T_{eff1} and T_{eff2} having the temperature of the first point of the recursive method (for a constant heat capacity is the geometric mean), then, the Carnot cycle constructed in this way has the same efficiency than the internal Carnot cycle in the constant heat capacity case, since in the *MW* case $T = T'$, then

$$\eta_{MW} \approx 1 - \frac{T_{eff2}}{T_{eff1}} = 1 - \frac{\sqrt{T_c T}}{\sqrt{T' T_h}} = 1 - \sqrt{\tau} = 1 - \frac{T_{cw}}{T_{hw}} = \eta_{MP} \tag{5}$$

and in the ME_r case $T = T'\left(\frac{1+\tau}{2}\right)$

$$\eta_{ME_r} \approx 1 - \frac{T_{eff2}}{T_{eff1}} = 1 - \frac{\sqrt{T_c T}}{\sqrt{T' T_h}} = 1 - \sqrt{\frac{\tau(1+\tau)}{2}} = 1 - \frac{T_{cw}}{T_{hw}} = \eta_{ME}. \tag{6}$$

Then, the correspondence between both cycles is obtained. In fact, it was established by Wang and Tu [7], for a CA-type engine, that the operation of the *MP* regime is equivalent to minimum irreversible entropy production in each finite-time "isothermal" process. Notice that, as mentioned above, for $n = 0$ one uses the geometric mean, however, for different values of n one should use a different mean. In Fig. 5 we show two cases of heat capacities ($n = 2$ and $n = -3/2$) and plot the efficiencies obtained with the effective temperatures (continuous lines), the reversible Otto-type cycle (dashed lines) and the endoreversible engines' efficiencies (dotted lines). In each case an inset shows how by adding points into the irreversible path one can approximate to the reversible cycle (shaded area).

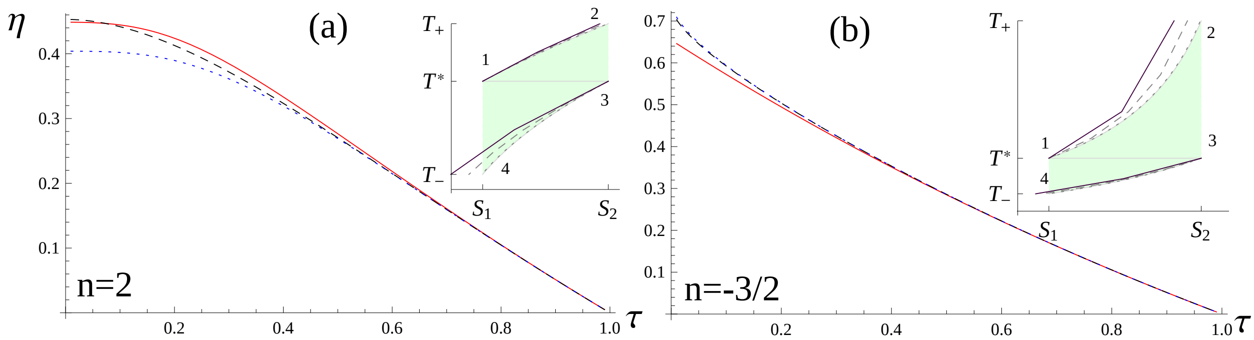

Figure 5: Comparison between three efficiencies: the reversible JB-type cycle efficiency (dashed line); its irreversible approximation obtained from the procedure described in the text (continuous line) and that corresponding to the FTT case (dotted line) for (a): n=2 and (b): n=-3/2. In each inset there is a JB cycle (colored region) with the condition of *MW* $T = T'$). Three paths are shown for the processes $1 \to 2$ and $3 \to 4$; one built with one auxiliary reservoir (thin line), another with three auxiliary reservoirs and the third one with 62 auxiliary reservoirs, notice that the more auxiliary reservoirs, the better the approximation to the reversible process.

Another way to see the endoreversible hypothesis has to do with the replacement of the Carnot cycle with another reversible cycle. In a more recent work [8] have made a molecular dynamic study of an ideal gas inside a 2-dimensional piston moving at constant velocity $\pm u$. By means of this velocity and with the help of a thermalizing wall it is possible to reproduce an endoreversible behavior. This was also discussed by Izumida and Okuda in Ref. [9].

It can be proven that the entropy change of the cycle described by the gas has zero entropy generation, however, the piston has an irreversible behavior and one can optimize power output and the ecological function.

From these two examples, it is possible to see that there is a flexibility in the endoreversible hypothesis that could play a role in possible extensions of the application of the endoreversible heat engine model.

In Ref. [10] we showed that there is an equivalence between the named external loss factors of the Otto and Joule-Brayton-type cycles and those of the Carnot like endoreversible engines. Those loss factors account for the deviations between the efficiencies of real devices, the reversible realization of such processes and from the Carnot cycle, all of them operating with the same heat input. In addition, in Ref. [5] we have shown that the so called *g*-function, which is the ratio between power output and dissipation of the HE is almost the same for the ratio between work and the so-called "geometrical dissipation" for a reversible cycle, being both quantities a general property independent of heat transfer laws in the former case and of heat capacity in the latter case. This reinforce the idea that the reversible cycles and the Carnot core of the CA HE share a variety of properties that could be explored by means of the flexibility of the endoreversible hypothesis.

REFERENCES

[1] F. L. Curzon and B. Ahlborn, Efficiency of a Carnot engine at maximum power output, *Am. J. Phys* vol. 43, 22, 1975.

[2] Julian Gonzalez-Ayala and L. A. Arias-Hernandez and F. Angulo-Brown, Connection between maximum-work and maximum-power thermal cycles, *Phys. Rev. E* vol. 88, 052142, 2013.

[3] F. Angulo-Brown, An ecological optimization criterion for finite-time heat engines, *J. Appl. Phys.* vol. 69, 7465-7469, 1991.

[4] A. Calvo Hernández, A. Medina, J. M. M. Roco, J. A. White, and S. Velasco, Unified optimization criterion for energy converters *Phys. Rev. E* vol. 63, 037102, 2001.

[5] Julian Gonzalez-Ayala and L. A. Arias-Hernandez and F. Angulo-Brown, A graphic approach to include dissipative-like effects in reversible thermal cycles, sent to *EPJB*.

[6] J. Anacleto and J. M. Ferreira, Minimizing the generation of entropy: which sequence of reservoirs to choose?, *Eur. J. Phys.* vol. 31, L1-L4, 2010.

[7] Y. Wang and Z. C. Tu, Efficiency at maximum power output of linear irreversible Carnot-like heat engines, *Phys. Rev. E* vol. 85, 011127, 2012.

[8] David A. Rojas-Gamboa, Juan I. Rodrguez, Julian Gonzalez-Ayala and F Angulo-Brown, On the ecological effciency of Finite-time thermodynamics: A Molecular Dynamics study, manuscript in preparation.

[9] Y. Izumida and K. Okuda, Molecular kinetic analysis of a finite-time Carnot cycle, *Europhys. Lett.* vol. 83, 60003, 2008.

[10] J Gonzalez-Ayala, F Angulo-Brown, A Calvo Hernández and S Velasco, On reversible, endoreversible, and irreversible heat device cycles versus the Carnot cycle: a pedagogical approach to account for losses, *Eur. J. Phys.* vol. 37, 045103, 2016.