NEWS ON NON-BOLTZMANNIAN THERMOSTATISTICAL SYSTEMS

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ABSTRACT

The necessary and sufficient microscopic conditions for validity of Boltzmann-Gibbs (BG) statistical mechanics remains until today an unsolved and surely challenging problem. This is essentially what is sometimes referred to as the 'Boltzmann program'. The most usual line of thinking for say isolated classical many-body Hamiltonian systems goes like this: If the interactions, typically two-body ones, are short-range in some relevant manner, then its dynamics are strongly chaotic (i.e., its maximal Lyapunov exponent is positive), hence it is mixing, hence it is ergodic. Ergodicity in such circumstances is typically understood to imply, for times long enough for having achieved the so called 'thermal equilibrium' collective stationary state. This is the basic scenario that justifies the assumption which uniquely and completely characterizes the microcanonical ensemble, namely the celebrated Boltzmann hypothesis of equiprobability of all possible microstates of an isolated system. It is however very important to realize that the full and rigorous logical-mathematical links of all these steps still remain today as an open very challenging problem. The full comprehension of all these conditions for the microcanonical ensemble, as well as its consequences for the canonical ensemble (when the system is in thermal equilibrium with a large thermostat), would constitute the complete solution of the magnificent Boltzmann program. In particular, we would then know when precisely should we expect the use of the BG factor to be mathematically legitimate.

Naturally, some of these issues have been tackled in one way or another by various physicists since Boltzmann formulated his impressive theory in the 1870's. We may mention two enlightening comments, by Ettore Majorana and Enrico Fermi respectively.

Majorana wrote [1]: "This is mainly because entropy is an additive quantity as the other ones. In other words, the entropy of a system composed of several independent parts is equal to the sum of entropy of each single part. [...] Therefore one considers all possible internal determinations as equally probable. This is indeed a new hypothesis because the universe, which is far from being in the same state indefinitely is subjected to continuous transformations. We will therefore admit as an extremely plausible working hypothesis, whose far consequences could sometime not be verified, that all the internal states of a system are a priori equally probable in specific physical conditions. Under this hypothesis, the statistical ensemble associated to each macroscopic state A turns out to be completely defined."

Fermi wrote [2]: "The entropy of a system composed of several parts is very often equal to the sum of the entropies of all the parts. This is true if the energy of the system is the sum of the energies of all the parts and if the work performed by the system during a transformation is equal to the sum of the amounts of work performed by all the parts. Notice that these conditions are not quite obvious and that in some cases they may not be fulfilled. Thus, for example, in the case of a system composed of two homogeneous substances, it will be possible to express the energy as the sum of the energies of the two substances only if we can neglect the surface energy of the two substances where they are in contact. The surface energy can generally be neglected only if the two substances are not very finely subdivided; otherwise, it can play a considerable role."

On these and related bases, I proposed in 1988 [3] the possibility of generalizing BG statistical mechanics through the use of a nonadditive entropy characterized by an entropic index q, such that in the limit q=1, the additive BG entropy is recovered. This proposal is nowadays the subject of intensive activity. In the Bibliography at http://tsallis.cat.cbpf.br/biblio.htm many analytical, experimental, observational and computational studies can be found focusing on natural, artificial and social systems. Many theoretical predictions and its experimental verifications are today available as well. After concisely presenting some of them (granular matter, high-energy collisions of elementary particles, standard map, scale-free networks, for instance), we will focus on the discussion of some of the hypothesis described here above. Two illustrative cases may be presented for possible discussion: (i) The hypothesis of equal probabilities dramatically fails when analytically determining the value of the index q which guarantees thermodynamical extensivity for the nonadditive entropy of a large subsystem of an even (much) larger system at its fundamental state, with full quantum entanglement (this violation is consistent with the fact that the subsystem is in a statistical mixture, unlike the full system, which is in a pure state); (ii) The classical long-range-interacting many-body Hamiltonian models extending the coupled XY-rotators, the coupled Heisenberg-rotators and the Fermi-Pasta-Ulam coupled quartic-anharmonic oscillators systematically exhibit, in first-principle molecular dynamics, q-statistics with q different from one sensibly outside the region of physical parameters where the maximal Lyapunov exponent vanishes in the limit of very large number N of elements of the system (this fact, even if it is numerically and not analytically, kind of suggests that positive maximal Lyapunov exponent might be necessary but not sufficient for legitimating the use of the BG factor in classical systems whose interactions are short-ranged enough to guarantee ergodicity and finiteness of the BG partition function, but still not enough to guarantee everything else that BG statistical mechanics requires.

REFERENCES

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