POWER FROM SIMPLEST STEADY-STATE QUANTUM HEAT ENGINE

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ABSTRACT

This talk discusses some generic issues of obtaining constant power from the simplest (three-level) quantum heat engine. The models are fully quantum, whereas the validity of Markovian approximations and the corresponding master equations will be assumed for the convenience of analytic calculations.

Working fluid of the simplest abstract quantum heat engine can be a three-level system

$$|0\rangle, |E_c\rangle, |E_h\rangle$$

of excitation energies $E_c < E_h$. The transition $|0\rangle \to |E_c\rangle$ is being cooled by the cold bath, the transition $|0\rangle \to |E_h\rangle$ is being heated by the hot bath at temperatures $T_h > T_c$, respectively. It has long been been known [1-4] that a population inversion will be established between the levels $|E_c\rangle$ and $|E_h\rangle$ provided the following condition is satisfied:

$$\frac{T_c}{T_h} < \frac{E_c}{E_h}.$$

The population inversion corresponds to the negative temperature

$$T_e^- = \frac{E_h - E_c}{\frac{E_h}{T_h} - \frac{E_c}{T_c}} < 0.$$

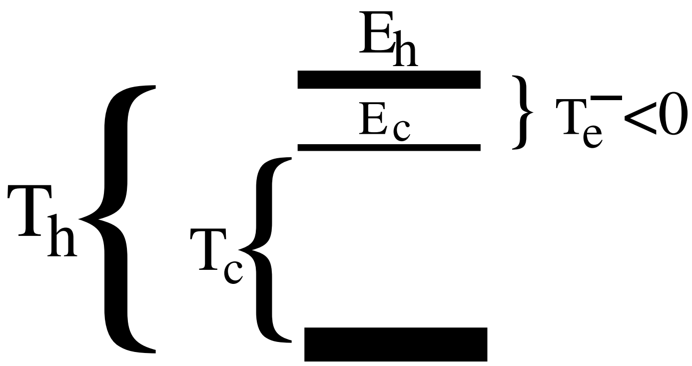

Figure 1: Simplest quantum heat engine is a three-level-system in contact with hot and cold heat baths, respectively. Equilibrium populations of the levels are (qualitatively) visualized by thicknesses of the levels. Population inversion, i.e.: negative effective temperature, develops for the middle+upper levels under the heat engine condition $T_c E_h < T_h E_c$. This population inversion serves as resource of work that we accumulate in the battery which should be a quantum system possessing a classical analogue.

We extract work from the engine by coupling a suitable battery to the effective two-level-system $|E_c\rangle, |E_h\rangle$. If this coupling is weak, we expect that population inversion is being maintained by the heat engine and the battery accumulates a macroscopic amount of energy. Moreover, we expect that a certain constant power can be extracted from the battery and a stationary working point can be maintained for it. We think of different options to charge a battery: (1) spin up a rotator, (2) excite an oscillator, (3) lift a weight, (4) generate voltage.

By macroscopic analogy, the typical battery is a mechanical flywheel to store kinetic energy. Macroscopic flywheel is a *rotator* (1) and we would consider its quantized version coupled to the engine. The calculations have not yet been done. A much simpler flywheel is the harmonic *oscillator* (2) of Hamiltonian $\hbar\omega_o \hat{c}_o^\dagger \hat{c}_o$, because it can be coupled to the engine on resonance

$$\hbar\omega_o = E_h - E_c$$

and assuming the simple coupling to $\hat{a}_e = |E_c\rangle\langle E_h|$:

$$i\hbar g \left(\hat{a}_e^\dagger \hat{c}_o - \hat{a}_e \hat{c}_o^\dagger \right).$$

Due to the population inversion, the engine is supplying quanta to the oscillator. We can out-couple useful work from the oscillator but the quantum and thermal fluctuations will go constantly increasing. We can reach a true stationary regime by active quantum control of the oscillator. This is not

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unusual if we note that a classical flywheel also needs a regulator to ensure stable operation under noisy conditions. A detailed model of quantum monitoring and feedback control of the oscillator has been presented in [5]. One would think that the simplest battery is a *weight* (3) being lifted by the engine against gravity. Interestingly, the coupling is less trivial than in case of the oscillator it was. Nonetheless the qualitative behavior of the battery is similar. The weight is constantly elevating, engine's power is being stored in constantly increasing potential energy of the weight. But the engine is constantly increasing the fluctuation of the height reached by the weight. Again, a stationary working point can only be ensured if we apply quantum control of the height of the weight. (Work is in preparation, by the same authors LDK.) As to the electric battery (4), it is qualitatively different since it consists of two infinite electric reservoirs in the simplest model. It is well possible that the steady-state operation with constant voltage and current can be obtained without the control, the details are subject to future investigations.

The mathematical model of the heat engine as well as of monitoring and control can be best based on master equations and Ito-stochastic master equations. A thermal bath of temperature T influences a two-level-system's density operator $\hat{\rho}$ via the standard thermalizing GKLS master equation [6,7]:

$$\frac{d\hat{\rho}}{dt} = \Gamma\left(\hat{a}\hat{\rho}\hat{a}^\dagger - \frac{1}{2}\{\hat{a}^\dagger\hat{a},\hat{\rho}\}\right) + \Gamma e^{-E/k_BT}\left(\hat{a}^\dagger\hat{\rho}\hat{a} - \frac{1}{2}\{\hat{a}\hat{a}^\dagger,\hat{\rho}\}\right)$$

where $\hat{a} = |0\rangle\langle E|$ and Γ is the decay rate. For $t \gg \Gamma$ the solution tends to the Gibbs state of the two-level-system at temperature T. Remarkably, if we derive the effective dynamics of the heat engine's 'output' two-level-system, it satisfies a similar master equation with the negative temperature T_e^-:

$$\frac{d\hat{\rho}_e}{dt} = \Gamma_e\left(\hat{a}_e\hat{\rho}_e\hat{a}_e^\dagger - \frac{1}{2}\{\hat{a}_e^\dagger\hat{a}_e,\hat{\rho}_e\}\right) + \Gamma_e e^{-E/k_BT_e^-}\left(\hat{a}_e^\dagger\hat{\rho}_e\hat{a}_e - \frac{1}{2}\{\hat{a}_e\hat{a}_e^\dagger,\hat{\rho}_e\}\right).$$

Obviously then, this equations drives the two-levels toward the population inverted state $|E_c\rangle\langle E_c| + e^{E/k_B|T_e^-|}|E_h\rangle\langle E_h|$. The above 'anti-thermalizing' master equation is the convenient starting point of subsequent coupling to the battery:

$$\frac{d\hat{\rho}_{eo}}{dt} = \Gamma_e\left(\hat{c}_e\hat{\rho}_{eo}\hat{c}_e^\dagger - \frac{1}{2}\{\hat{c}_e^\dagger\hat{c}_e,\hat{\rho}_{eo}\}\right) + \Gamma_e e^{-E/k_BT_e^-}\left(\hat{c}_e^\dagger\hat{\rho}_{eo}\hat{c}_e - \frac{1}{2}\{\hat{c}_e\hat{c}_e^\dagger,\hat{\rho}_{eo}\}\right) - g\left[\hat{a}_e^\dagger\hat{c}_o - \hat{a}_e\hat{c}_o^\dagger,\hat{\rho}_{eo}\right]$$

in interaction picture. That's an analytically tractable model of how the the GKLS dynamics of population inversion of the 'output' two-level-system of the engine (e) is charging the battery (o). Monitoring and control will require further (stochastic) refinement of the master equation.

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